Safety and Reliability – Bedford & van Gelder (eds)
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A logarithmic approach for individual risk: the safety-index

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ABSTRACT: Risk analyses can be undertaken to examine the required safety measures that are needed to realise complex projects near hazardous installation. When doing this risk analysis, the results have to be checked for risk acceptance criteria. In this paper, the three main criteria for risk acceptance criteria, which can be divided into individual risk, risk on a social basis and the economic criterion, are analysed and their interrelation is described. One of the relations between these criteria is the expected number of casualties. To quantify this expected number of casualties in term of economics, the expected numbers of casualties are taken into account by using monetary value per casualty. This paper discusses the variation of the monetary value per casualty. Furthermore, the acceptable level for societal risk is analysed for different countries. Finally, a new approach for the individual risk criterion on logarithmic scale, namely *the safety-index* is discussed in this paper. This paper describes a full derivation of the safety-index. Besides, on the basis of the safety-index, a dimensionless criterion for individual risk is proposed. The safety-index provides an effective tool for the assessment of individual risk dimensionless regarding the acceptance of risk.

1 INTRODUCTION

During the design phase of a complicated project, risk analyses can be undertaken to examine the required safety measures that are needed to realise such projects. When doing this risk analysis, the results have to be checked for risk acceptance criteria. If the results do not comply with these risk acceptance criteria, to be divided into criteria on an *individual* and on a *social* basis, extra measures can be taken to increase the level of safety. However, these risk acceptance criteria are different to each country. In order to take decisions for safety measures it is useful that the main criteria for risk acceptance criteria are analysed and their interrelation with economic considerations is described.

Moreover, the realisation of safety measures is related to investments. In this regard, economic considerations have to be taken into account when risk analysis is performed and measures are going to be taken. These economic considerations consists costs for investments and the economic risk. Considering these measures, the decision maker finds himself in a dilemma: which measure has to be given preference, the one that minimises the economic risk or the one that decreases the loss of human lives. Generally, in such analyses it comes down to the fact that human risks e.g. expected number of casualties are also transformed into monetary terms. This paper will present the variation of the monetary value per casualty.

Another complexity during the design phase of complicated projects is the transparency of the risk acceptance criteria for not-scientists e.g. municipalities. Considering these criteria, it is a difficulty for the decision maker to understand these criteria. The individual risk, which is one of these criteria, is traditionally depicted as contours on a – two-dimensional – map [Ale et al., 1996]. When depicting such risk contours, only the probability of a person is given, who permanently is present at a certain location in the vicinity of a hazardous activity will be killed as a consequence of an accident with that activity. However, these risk contours does not provides the acceptance of risk, which can be divided into the voluntariness and the direct benefit, of that person. Different participants in the exploitation phase require different demands and therefore have a different perception of safety. Therefore, it is recommendable to implement these risk contours including the voluntariness and the direct benefit of these participants.

Accordingly, in this paper, a new (dimensional) approach for the individual risk criterion on logarithmic scale, namely *the safety-index* is proposed [Suddle, 2002[A]]. This logarithmic approach is adapted from medical sciences and insurance policies [Boudier et al., 1985], which can be applied in building engineering and physical planning around hazardous installations and infrastructure with transport of hazardous materials to present safety results dimensionless and including

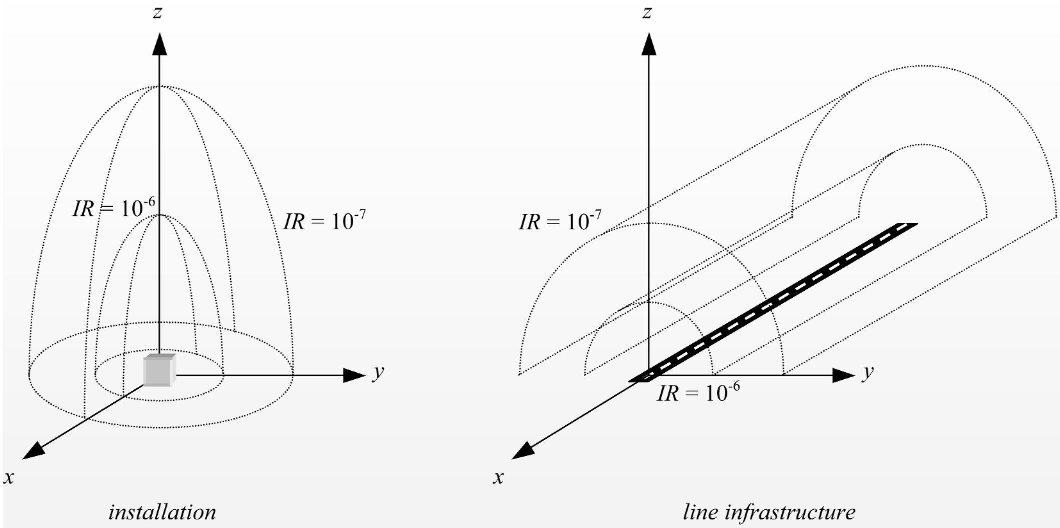

Figure 1. Two and three-dimensional individual risk contours for an installation and line infrastructure [Suddle et al., 2002].

personal acceptable level of risk. The formula of safety-index is applied to a line infrastructure case in which (individual) safety contours are depicted. This concept can be handy for policy makers and thus effective in risk communication.

2 RISK ACCEPTANCE AND DECISION-MAKING

Risk analysis is a method that can be used to examine the safety in objective terms. When doing this risk analysis, the results have to be checked for risk acceptance criteria. Criteria for accepting or rejecting the assessed risks include two related entities: the frequency of an undesired event and the consequences (casualties, monetary values, environmental values). In general, one may state that the higher the consequences, the lower the accepted probabilities are. In more detail, the acceptance limits for a given event may originate from three different angles [Vrouwenvelder et al., 2001]:

1. A comparison with other risks related to individual safety;
2. Societal aversion to big disasters, especially when many casualties are involved;
3. Economic considerations.

If the results do not comply with these risk acceptance criteria, measures can be taken to increase the required level of safety. However, these measurements have to be attractive in terms of economics. Moreover, these three aspects should be integrated and/or prioritised.

3 A SET OF RULES FOR THE ACCEPTABILITY OF RISKS

3.1 *Personally acceptable level of risk*

An overview of measures to express the individual risk is given by [Bedford & Cooke, 2001]. The smallest component of the social acceptable of risk is the personal cost-benefit assessment by the individual [Vrijling et al., 1998].

Individual risk (*IR*) is defined as the probability that a person who permanently is present at a certain location in the vicinity of an activity will be killed as a consequence of an accident with that activity. Usually, *IR* is expressed for a period of a year. It can be pictured both on two and three-dimensional [Suddle et al., 2002] map by connecting point of equal *IR* around a facility, the risk contours [Ale, 2002].

From a personally point of view, the probability of failure (a fatal accident) should meet the following requirement [Vrijling & Vrouwenvelder, 1997]:

$$P_{fi} \leq \frac{\beta_i \cdot 10^{-4}}{P_{d|fi}} \quad (1)$$

In which:

P_{fi} = probability of failure f as a result of an event i [year^{-1}];

$P_{d|fi}$ = probability of being killed if failure f as a result of an event i, occurs;

β_i = the policy factor that varies with the degree of voluntariness with which an activity i is undertaken and with the benefit perceived. It ranges from 100, in case of complete freedom of

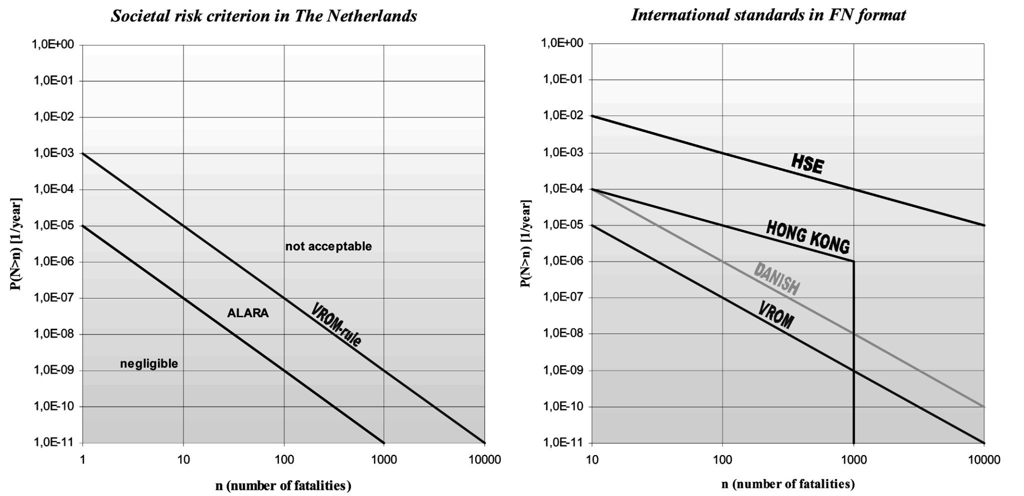

Figure 2. FN curves where $1 - F_N(n) = P(N > n$ in one year) is illustrated in The Netherlands (left) and some international FN standards (right).

Table 1. Personal risks in Western countries, deduced from the statistics of causes of death and the number of death and the number of participants per activity [Vrijling et al., 1998].

Probability of dying per year	Statistics of causes of death	Acceptance of risk		Policy factor
		high ↑	yes ↑	
10^{-2}	Mountaineering	voluntariness	direct benefit	$\beta_i = 100$
	illness			$\beta_i = 10$
10^{-4}	motoring			$\beta_i = 1$
	flying			$\beta_i = 0{,}1$
10^{-6}	factory			$\beta_i = 0{,}01$
		↓ low	↓ no	

choice like mountaineering, to 0,01 in the case of an imposed risk without any perceived direct benefit;

10^{-4} = statistical probability of dying per year of young people [year^{-1}].

3.2 *Socially acceptable level of risk*

Societal risk (*SR*) is defined as the probability that in an accident more than a certain number of people are killed. Societal risk usually is represented as a graph in which the probability or frequency *F* is given as a function of *N*, the number killed. This graph is called the *FN* curve. A mathematical expression in the case of a straight *FN* curve (on log-log-scale) can be presented as a combination of [Vrijling et al., 1998] and [Vrouwenvelder et al., 2001]:

$$1 - F_N(n) \le \frac{C_i}{n^\gamma} \quad \text{for all } n \ge 10 \tag{2}$$

$$1 - F_N(n) = P(N > n) \tag{3}$$

where

$$C_i = \left[\frac{\beta_i \cdot 100}{k \cdot \sqrt{N_A}}\right]^2 \tag{4}$$

In which:

C_i = the (imaginary) acceptable probability for $n = 1$;
$1 - F_N(n)$ = frequency of more than n fatalities [year^{-1}];
N = the number of people being killed in one year in one accident;
n = number of fatalities in one year in one accident;
N_A = the independent locations;
γ = the slope of the *FN* curve, also called the risk aversion factor [Vrijling & Gelder, 1997]; the value of γ ranges from 1 to 2;
k = the risk aversion factor; the value of k mostly is 3.

A standard with a steepness of $\gamma = 1$ is called risk neutral. If the steepness $\gamma = 2$, the standard is called risk averse. In this case larger accidents are weighted more heavily and accepted with a relatively lower probability. Some international *FN* standards are given in figure 2 (right) [Jonkman et al., 2002]. In contrast to other countries, the societal risk criterion in The Netherlands is much stringent. Hence, it is not remarkable that the result some safety studies does not comply with the Dutch criteria (VROM-rule), while for instance in other countries, they do comply.

In general, the *FN* curve indicates the border between "acceptable" and "unacceptable" in a diagram with probability on one axis and the number of casualties on the other. It is quite customary to have two *FN* curves as indicated in figure 2 (left):

- One curve representing an upper limit above which activities or situations are not acceptable;
- Another curve representing a lower limit below which no further risk reductions are necessary.

In figure 2 the societal risk criterion in The Netherlands, also called the VROM-rule, is illustrated. In the area in between risk reducing measures should be considered and judged on an economical basis. Between these levels, it is required to reduce risks to levels as "as low as reasonable achievable" (ALARA) that is, until the costs of further measures would be grossly disproportionate to the benefit gained.

3.3 *Economic criteria*

According to [Vrouwenvelder et al., 2001], the third acceptance creation can be schematised as a mathematical-economic decision problem by expressing both investments and all consequences of the disaster in terms of money (assuming a given period of time).

Besides, it may be suggested that a measure with less human risk is more expensive than a one with gigantic risk. To balance these measures an economic creation is required. It means that the most economical solution from all alternatives that are allowable from the human safety point of view. Mathematically it comes down to [Vrouwenvelder et al., 2001]:

Minimise:

$$C_{tot} = C_0(y) + \sum_{j=1} \frac{P_{Fj} \cdot \{C_j + \alpha \cdot E(N_d \mid F)\}}{(1+r)^j}$$

$$\text{Conditional upon: } P_{fi} \leq \frac{\beta_i \cdot 10^{-4}}{P_{d|fi}} \text{ or} \qquad (5)$$

$$S = \log \frac{\beta_i \cdot 10^{-4}}{P_{fi} \cdot P_{d|fi}} \geq 0 \text{ (see section 4)}$$

$$1 - F_N(n) \leq \frac{C_i}{n^{\gamma}}$$

In which:

C_{tot} = total costs;
$C_0(y)$ = the investment in a safety measure;
j = the number of the year;
r = real rate of interest;
C_j = damage cost in year j;
y = decision parameter;
α = monetary value per casualty;
$E(N_d \mid \text{F})$ = expected number of casualties given a failure; $E(N_d) = P_{fi} \cdot P_{d|fi} \cdot N_{pi}$; $E(N_d \mid \text{F}) = P_{d|fi} \cdot N_{pi}$;
N_{pi} = number of participants in activity i;
$P_{Fj}(y)$ = the failure in year j.

One should realise that $P_{Fj}(y)$ denotes the failure exactly in year j, that is not in any year before or later. The term C_j includes all costs after failure (also called the material losses): it includes direct damage, cost of repair, but also future failure costs of the repaired structure (if any).

3.4 *Monetary value per casualty*

Most decision makers prefer to treat the economic and human safety criteria completely separated. In that case, the value of $\alpha = 0$; this is the creation fully compatible to the approach of a purely economic decision problem. Still, there are some decision makers who compare the advantage of safety measures in comparison with economic investments. Having this in mind, it might be better to assess some amount of money to the event for death or injury. For this purpose the amount for material damage is increased with the monetary value per casualty multiplied by the expected number of death (as presented in formula 5). The monetary value per casualty depends on factors such as Willingness To Pay (WTP), Willingness To Accept compensation (WTA), voluntariness, and responsibility [Jones-Lee & Loomes, 1995]. According to the Environmental Protection Agency the value of a citizen in the US is approximately €5,600,000. = . It may be concluded from [@1], that these values result in a wide range. According to [Vrouwenvelder et al., 2001] a reasonable value seems €1,000,000. = . Another method to determine this value is the so called Life Quality Index (LQI) (see [Lind, 1994]). The values per casualty can be summarised in table 2.

Table 2. Investments in Risk Reduction, per nominal lives saved [University of East Anglia in 1998].

Theoritical Evaluations	Value for α [€ per person]
Human capital calculations	300,000
Willingness to pay (hypothetical)	1,600,000
Road Safety (UK, 1987)	500,000
Cost of medical procedures for comparison (real)	2,000–300,000

4 THE SAFETY-INDEX

4.1 *Introduction*

According to [Boudier et al., 1985], most decision makers prefer to present the risk results on a dimensionless scale. Therefore [Boudier et al., 1985] used a logarithm scale for presenting the individual risk dimensionless. This logarithmic scale is used in medical sciences and insurance policies [Boudier et al., 1985]. In this scale, *the unikohort*, is defined as the negative logarithm of individual risk for a period of 1 year:

$$U = -\log P_{fi} \cdot P_{d|fi} \tag{6}$$

In which:
U = unikohort.

Note that this formula does not contain a correction factor for risk acceptance. In order to integrate the factor for risk acceptance we can analyse the individual risk. Considering the acceptable level for individual risk, one may remark that improvements in the level of risk do make sense, when risk increases with a factor ten [Suddle, 2002]. Similarly, a decrease of risk with a factor ten is a remarkable worsening. The factor ten suggests a logarithmic scale with base 10. Obviously, societal risk is displayed on a (double) logarithm scale. Individual risk, as early mentioned in this paper, can be determined by risk analysis and subsequently checked for the risk acceptance criteria. Writing formula (1) in another configuration gives:

$$IR = P_{fi} \cdot P_{d|fi} \leq \beta_i \cdot 10^{-4} \tag{7}$$

In which:
IR = the individual risk (as mentioned before).

Formula (7) can be written as:

$$\frac{P_{fi} \cdot P_{d|fi}}{\beta_i \cdot 10^{-4}} \leq 1 \tag{8}$$

Though the check in formula (8) is dimensionless, yet, it presents the ratio of individual risk and the risk acceptance criterion, which is hardly interesting. This check is rather attractive if this is done on a base of a (negative) logarithmic scale. By considering the usual definition of risk, a possible standard or a scale for safety in terms of individual risk can be given by:

$$S = -\log \frac{P_{fi} \cdot P_{d|fi}}{\beta \cdot 10^{-4}} \tag{9}$$

In which:
S = the safety-index (Dutch: Veiligheidsmaat (see [Suddle, 2002])) [-];

This introduces a new definition for the individual risk; *the safety-index*. In this formula (9), the referential level for acceptable safety is the level that (just) complies with the acceptability of individual risk. Eliminating the minus before the logarithm gives the following:

$$S = \log \frac{\beta_i \cdot 10^{-4}}{P_{fi} \cdot P_{d|fi}} \tag{10}$$

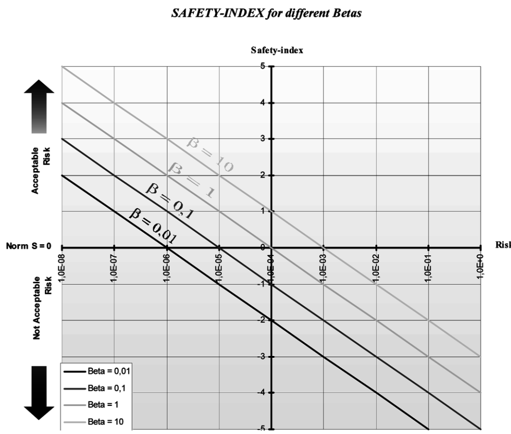

Figure 3. Safety-index versus individual risk by different β_is.

The result of the safety-index S is a particular number. In fact, a distinction can be made for the three following situations:

1. $S < 0$ The computed safety/risk does not comply with the level of risk acceptance. The more the risk exceeds the norm ($\beta_i \cdot 10^{-4}$), the smaller is the safety-index, and the unsafe is the activity. (A decrease of the safety-index with one means that the risk increases with one level);
2. $S = 0$ The computed safety/risk complies with the level of risk acceptance;
3. $S > 0$ The computed safety/risk complies largely with the level of risk acceptance. (An increase of the safety-index with one means that the risk decreases with one level).

It can be assumed that one strives for situation 2 and 3, thus $S \geq 0$. Combined with formula 10, this results in the norm for safety in terms of individual risk:

$$S = \log \frac{\beta_i \cdot 10^{-4}}{P_{fi} \cdot P_{d|fi}} \geq 0 \tag{11}$$

For decision maker it is attractive to present safety results of in terms of individual risk, formula (11) can be used rather than formula (1). Note that different safety- index cannot be summed up. If one likes to present risk results on a dimensionless scale, one has to sum up different individual risks and than to take the logarithm of it.

The result of the safety-index depends on the individual risk and the β_i. Table 3 and the diagram represent the relation between the safety-index and individual risk for different β_is. This model enables safety in terms of individual risk can be quantified and can be checked directly for the limits of risk acceptance for

Table 3. Safety-index versus individual risk for different β_is.

Individual Risk	Safety-index			
$IR = P_{fi} \cdot P_{d\|fi}$	$\beta_i = 0{,}01$	$\beta_i = 0{,}1$	$\beta_i = 1$	$\beta_i = 10$
10^{-2}	-4	-3	-2	-1
10^{-3}	-3	-2	-1	0
10^{-4}	-2	-1	0	1
10^{-5}	-1	0	1	2
10^{-6}	0	1	2	3
10^{-7}	1	2	3	4
10^{-8}	2	3	4	5
10^{-9}	3	4	5	6

Not acceptable

Acceptable

individual risk. This instrument provides an effective tool for determining the effects of the safety(-index) on safety-measures if the design limit is based upon the individual risk.

Furthermore, the next limit is applicable:

$$\lim_{IR\downarrow 0}(S) = \infty \tag{12}$$

With other words: if there is no risk, the safety (-index) will approach infinite.

4.2 *Unikohort and the safety-index*

If the unikohort is compared to the safety-index, there is no correction factor for acceptance of risk taken into account. In order to deduce the safety-index from the unikohort, the risk acceptance factor must be integrated into the unikohort. The correction factor for acceptance of risk can be given by:

$$A = -\log \beta_i \cdot 10^{-4} \tag{13}$$

In which:

A = correction factor for acceptance of risk.

In order to compute an index for individual safety including the acceptability, one can deduce the correction factor for acceptance of risk:

$$S = U - A \tag{14}$$

$$S = -\log P_{fi} \cdot P_{d|fi} - (-\log \beta_i \cdot 10^{-4}) \tag{15}$$

$$S = \log \beta_i \cdot 10^{-4} - \log P_{fi} \cdot P_{d|fi} \tag{16}$$

$$S = \log \frac{\beta_i \cdot 10^{-4}}{P_{fi} \cdot P_{d|fi}} \tag{17}$$

This formula is exact the same as formula (10).

Table 4. Different approach for local residents near infrastructure and car drivers at the infrastructure.

Local residents near infrastructure	Car drivers at the infrastructure
$P_{fi} = 10^{-5}$ [year^{-1}];	$P_{fi} = 10^{-5}$ [year^{-1}];
$P_{d\|fi} = 0.99$ [−];	$P_{d\|fi} = 1$ [−];
$\beta_i = 0.01$ (involuntary activity)	$\beta_i = 1$ (voluntary activity)
$IR = 9.9 \cdot 10^{-6} \cong 10^{-5}$ [year^{-1}]	$IR = 10^{-5}$ [year^{-1}]

4.3 *Example*

Formulas (10) and (11) provide an effective tool, particularly for decision makers, which can be presented in the following example in which the individual risk and the safety-index is computed and compared for local residents near infrastructure and car drivers at the infrastructure. Suppose the following situation in which an accident occurs on the infrastructure with a probability of 10^{-5} [year^{-1}]:

The safety-index S for this example can be computed with formula (10), which is for local residents near infrastructure:

$$S = \log \frac{0{,}01 \cdot 10^{-4}}{10^{-5} \cdot 0.99} = \log \frac{10^{-6}}{9.9 \cdot 10^{-6}} = \log 0.11 \cong -1 \tag{10a}$$

The safety-index S for car drivers at the infrastructure is:

$$S = \log \frac{1 \cdot 10^{-4}}{10^{-5} \cdot 1} = \log \frac{10^{-4}}{10^{-5}} = \log 10 = 1 \tag{10b}$$

The result of the safety-index S is a particular number, which is respectively −1 and 1 for local residents near infrastructure and car drivers at the infrastructure. Though the individual risk IR for both local residents near infrastructure and car drivers is almost the same (10^{-5} year^{-1}), the safety-index S has a different

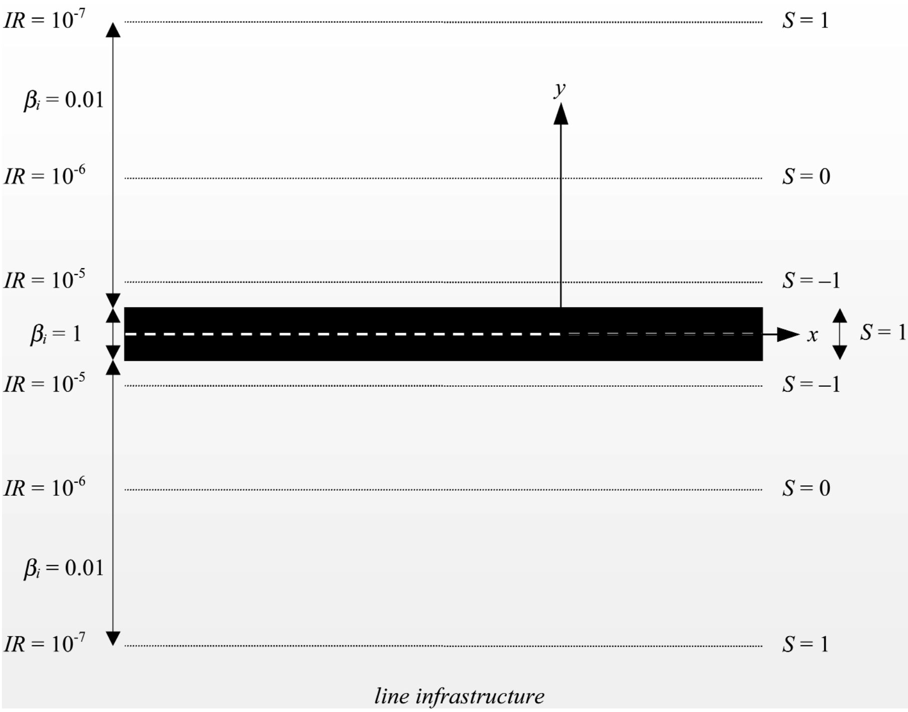

Figure 4. Individual risk contours (left) and safety contours (right).

value for both. This comes down to the fact that the safety (in term of individual risk) for local residents near infrastructure is insufficient, because the limit for acceptance of risk is exceeded. Accordingly, people present in the neighbour of the infrastructure, especially within the 10^{-5} risk contour, will accept less risk than the car drivers.

The phenomena of the safety in terms of individual risk can be illustrated by connecting the points with the same safety-index yields an iso-safety contour, which is related both to the individual risk and the acceptance of risk. Figure 4 visualizes the idea of individual risk contours and the safety contours. In this figure, the policy factor β_i is given, which represents the risk acceptance as mentioned in table 1.

It can be noted that just outside the boundary of the infrastructure the safety-index is below zero ($S < 0$). Furthermore it can be seen that the individual risk decreases results in the increase of the safety(-index).

5 CONCLUSIONS

This paper contributes to the he transparency of the risk acceptance criteria. As a consequence, the interrelation between three main criteria for risk acceptance criteria, which can be divided into individual risk, risk on a social basis and the economic criterion, is described. It may be concluded, the new approach for the individual risk criterion on logarithmic scale, namely *the safety-index* is handy for policy makers and therefore effective in risk communication. Thus, this logarithmic approach for the individual risk criterion partly adapted from medical science and insurance policies can be applied in civil engineering to present risk results on a dimensionless scale.

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